Worksheet for 2021-10-06

Conceptual questions

These problems are taken from the announcement that Prof. Sethian sent out on 10/5.

Question 1. Can any two arbitrary (nice) functions comprise the partial derivatives f_x, f_y of another function, or are there some restrictions?

Question 2. When doing Lagrange multipliers for f subject to $g = 0$, is it necessary that ∇f and ∇g point in the same direction for an extremum, or is opposite directions fine as well?

Question 3. Suppose P is a point where $f_x(P) = f_y(P) = 0$, $f_{xx}(P) > 0$, $f_{yy}(P) > 0$, and $f_{xy}(P) > 0$. Does this mean P is a local minimum of f?

Question 4. Suppose P is a point where $f_x(P) = f_y(P) = 0$ and for *every* unit vector $\mathbf{u}$,

$$(D_{\mathbf{u}}(D_{\mathbf{u}} f))(P) > 0.$$

Does this mean P is a local minimum of f? The parentheses in this problem were originally misplaced.

Question 5. Let H be the zero level set of a function $f(x, y)$. Can H intersect itself? If so, give an example. What happens if you try to compute a tangent line to H at such a point using the "gradient = normal" method?

Question 6. In Lagrange multipliers, we might consider the problem:

> Maximize the function $f(x, y, z)$ subject to the constraint that $g(x, y, z) = 0$.

Suppose the maximum occurs at (a, b, c) and $f(a, b, c) = 12$.

Now consider the other problem

> Maximize the function $g(x, y, z)$ subject to the constraint that $f(x, y, z) = 12$.

Is the point (a, b, c) also the answer to this problem?